WEEKLY TEST TYJ -1 TEST - 34 R
SOLUTION Date 05-01-2020

[PHYSICS]

1. **(a)** It is given that energy remains the same.

Hence, $E_A = E_B$

Energy $\propto a^2n^2 \Rightarrow \frac{a_B}{a_A} = \frac{n_A}{n_B}$ $\quad$ ($\because$ energy is same)

$$\therefore \quad \left(\frac{a_A}{a_B}\right)^2 = \left(\frac{n_B}{n_A}\right)^2$$

Given, $n_A = n, n_B = \frac{n}{8}$

$$\therefore \quad \frac{a_A}{a_B} = \frac{n/8}{n} = \frac{1}{8} \quad \Rightarrow a_B = 8a_A = 8a$$

2. **(d)** The frequency of note emitted by the wire,

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

m = mass m per unit length of wire and T = tension, and l = length of wire.

$$\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$$

Given, $T_1 = 10$ N, $n_1 = n$, and $n_2 = 2n$

$$\Rightarrow \quad \frac{n}{2n} = \sqrt{\frac{10}{T_2}} \quad \Rightarrow \quad T_2 = 10 \times 4 = 40 \text{ N}$$

3. **(c)** Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

Path difference $\Delta = \frac{\lambda}{2\pi} \times \phi = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$

4. **(a)** The apparent change in the frequency of the source due to relative motion between source and observer is known as Doppler's effect. The perceived frequency (n') when listener is static and source is moving away is given by

$$n' = n\left(\frac{v}{v+v_s}\right)$$

where n is frequency of source, v is velocity of sound and v_s is velocity of source.

Putting $v = 330$ m/s, $v_s = 30$ m/s, $n = 800$ Hz.

$$n' = 800 \times \left(\frac{330}{330+30}\right)$$

$$n' = 733.3 \text{ Hz}$$

In the limit when speed of source and observer is much lesser than that of sound v_1, the change in frequency becomes independent of the fact whether the source is moved or the detector.

5. **(b)** The velocity of sound is given by $v = \sqrt{\frac{\gamma P}{\rho}}$

where P is pressure, ρ is density and γ is adiabatic constant.

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{4}{1}} = 2:1$$

6. **(b)** Compare with $y = a \sin(\omega t - kx)$

We have $k = \frac{2\pi}{\lambda} = 62.4 \quad \Rightarrow \quad \lambda = \frac{2\pi}{62.4} = 0.1$

7. **(b)** The frequency produced in a string of length l, mass per unit length m, and tension T is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Given $l_1 = 50$ cm, $n_1 = 800$ Hz

and $n_2 = 1000$ Hz

$n_1 l_1 = n_2 l_2$

$\Rightarrow$ $800 \times 50 = 1000 \times l_2$

$\Rightarrow$ $l_2 = 40$ cm

8. **(d)** Points B and F are in same phase as they are λ distance apart.

9. **(c)** Water waves are transverse as well as longitudinal in nature.

10. **(d)** Fundamental frequency of open organ pipe $=\dfrac{v}{2l}$

Frequency of third harmonic of closed pipe $=\dfrac{3v}{4l}$

$\therefore \quad \dfrac{3v}{4l}=100+\dfrac{v}{2l}$

$\Rightarrow \quad \dfrac{3v}{4l}-\dfrac{2v}{4l}=\dfrac{v}{4l}=100 \Rightarrow \dfrac{v}{2l}=200\text{ Hz}$

11. **(a)** $dB=10\log_{10}\left[\dfrac{I}{I_0}\right]$,

where $I_0=10^{-12}\text{ wm}^{-2}$

Since $40=10\log_{10}\left[\dfrac{I_1}{I_0}\right] \Rightarrow \dfrac{I_1}{I_0}=10^4$

Also, $20=10\log_{10}\left[\dfrac{I_2}{I_0}\right] \quad \Rightarrow \dfrac{I_2}{I_1}=10^2$

$\Rightarrow \quad \dfrac{I_2}{I_1}=10^{-2}=\dfrac{r_1^2}{r_2^2}$

$\Rightarrow \quad r_2^2=100r_1^2 \quad \Rightarrow r_2=10\text{ m}$

12. **(b)** Given $\dfrac{I_1}{I_2}=\dfrac{4}{1}$

We know $I \propto a^2$

$\therefore \quad \dfrac{a_1^2}{a_2^2}=\dfrac{I_1}{I_2}=\dfrac{4}{1}$ or $\dfrac{a_1}{a_2}=\dfrac{2}{1}$

$\therefore \quad \dfrac{I_{max}}{I_{min}}=\dfrac{(a_1+a_2)^2}{(a_1-a_2)^2}=\left(\dfrac{2+1}{2-1}\right)^2$

$=\left(\dfrac{3}{1}\right)^2=\dfrac{9}{1}$

Therefore, difference of loudness is given by

$$L_1-L_2=10\log\frac{I_{max}}{I_{min}}=10\log(9)$$

$$=10\log 3^2=20\log 3.$$

13. **(a)** When the source is coming to the stationary observer,

$$n' = \left(\frac{v}{v - v_s}\right)n \quad \text{or} \quad 1000 = \left(\frac{350}{350 - 50}\right)n$$

or $n = (1000 \times 300/350)$ Hz

When the source is moving away from the stationary observer.

$$n'' = \left(\frac{v}{v + v_s}\right)n$$

$$= \left(\frac{350}{350 + 50}\right)\left(\frac{1000 \times 300}{350}\right)$$

$$= 750 \text{ Hz}$$

14. **(c)** Fundamental frequency of closed pipe

$$n = \frac{v}{4l} = 220 \text{ Hz} \Rightarrow v = 220 \times 4l$$

If 1/4 of the pipe is filled with water then remaining length of air column is $\frac{3l}{4}$

Now fundamental frequency $= \frac{v}{4\left(\frac{3l}{4}\right)} = \frac{v}{3l}$ and

First overtone = 3 × fundamental frequency

$$= \frac{3v}{3l} = \frac{v}{l} = \frac{220 \times 4l}{l} = 880 \text{ Hz}$$

15. **(c)** $f \; \alpha \sqrt{T}$

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\Rightarrow \quad \Delta f = \frac{202}{2} \times \frac{1}{101} = 1$$

16. **(d)** $\frac{v_1}{v_2} = \frac{28}{27}$

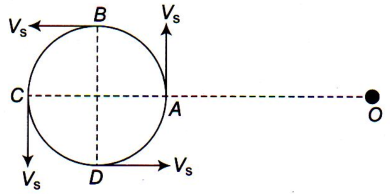
$$v_1 - v_2 = 3 \text{ or } \frac{28}{27}v_2 - v_2 = 3$$

$$v_2 = 27 \times 3 \text{ Hz} = 81 \text{ Hz}$$

or $v_1 = v_2 + 3 = (81 + 3)$Hz

or $v_1 = 84$ Hz

17. **(d)** Frequency heard by the observed will be maximum when the source is in the position D. In this case, source will be approaching towards the stationary observer, almost along the line of sight (as observer is stationed at a larger distance).



Similarly, frequency heard by the observer will be minimum when the source reaches at the position B. Now, the source will be moving away from the observer.

$$n_{\text{min.}} = \frac{v}{v+v_s} \times n = \frac{330}{330+1.5\times 20} \times 440$$

$$= \frac{330 \times 440}{360} = 403.3 \text{ Hz}$$

18. (d) When pulse is reflected from a rigid support, the pulse is inverted both lengthwise and sidewise

19. (c) Here $A = 0.05m$, $\frac{5\lambda}{2} = 0.025 \Rightarrow \lambda = 0.1m$

Now standard equation of wave

$$y = A\sin\frac{2\pi}{\lambda}(vt - x) \Rightarrow y = 0.05\sin 2\pi(33t - 10x)$$

20. (d) Intensity $\propto a^2\omega^2$

here $\frac{a_A}{a_B} = \frac{2}{1}$ and $\frac{\omega_A}{\omega_B} = \frac{1}{2} \Rightarrow \frac{I_A}{I_B} = \left(\frac{2}{1}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{1}$

[MATHEMATICS]

41. (c) $\sin^2\theta + \text{cosec}^2\theta = (\sin\theta + \text{cosec}\theta)^2 - 2\sin\theta\,\text{cosec}\theta$

$= (2)^2 - 2 = 4 - 2 = 2$, since $(\sin\theta + \text{cosec}\theta) = 2$.

42. (d) We have, $x + \frac{1}{x} = 2\cos\alpha$

$x^2 + \frac{1}{x^2} + 2 = 4\cos^2\alpha$.

$x^2 + \frac{1}{x^2} = 4\cos^2\alpha - 2$,

$x^2 + \frac{1}{x^2} = 2(2\cos^2\alpha - 1) = 2\cos 2\alpha$

Similarly $x^n + \frac{1}{x^n} = 2\cos n\alpha$.

43. (a) As given

$\frac{1}{\tan\theta} + \tan\theta = m \Rightarrow 1 + \tan^2\theta = m\tan\theta$

$\Rightarrow \sec^2\theta = m\tan\theta$ .....(i)

and $\sec\theta - \cos\theta = n \Rightarrow \sec^2\theta - 1 = n\sec\theta$

$\Rightarrow \tan^2\theta = n\sec\theta$

$\Rightarrow \tan^4\theta = n^2\sec^2\theta = n^2.m\tan\theta$ {by (i)}

$\Rightarrow \tan^3\theta = n^2 m$, $(\because \tan\theta \neq 0)$

$\Rightarrow \tan\theta = (n^2 m)^{1/3}$ .....(ii)

Also, $\sec^2\theta = m\tan\theta = m(n^2m)^{1/3}$ {by (i) and (ii)}

$\therefore$ Using the identity $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow m(mn^2)^{1/3} - (n^2m)^{2/3} = 1$

$\Rightarrow m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$.

44. (d) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1$,

$(\because -1 \le \sin x \le 1)$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2} \Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$.

45. (a) We know, $\sin 22\frac{1^\circ}{2} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

and $\cos 22\frac{1^o}{2} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\therefore \left(1+\cos 22\frac{1^o}{2}\right)\left(1+\cos 67\frac{1^o}{2}\right)\left(1+\cos 112\frac{1^o}{2}\right)\left(1+\cos 157\frac{1^o}{2}\right)$

$= \left(1+\frac{1}{2}\sqrt{2+\sqrt{2}}\right)\left(1+\frac{1}{2}\sqrt{2-\sqrt{2}}\right)\left(1-\frac{1}{2}\sqrt{2-\sqrt{2}}\right)\left(1-\frac{1}{2}\sqrt{2+\sqrt{2}}\right)$

$= \left[1-\frac{1}{4}(2+\sqrt{2})\right]\left[1-\frac{1}{4}(2-\sqrt{2})\right]$

$= \frac{(4-2-\sqrt{2})(4-2+\sqrt{2})}{16}$

$= \frac{(2-\sqrt{2})(2+\sqrt{2})}{16} = \frac{4-2}{16} = \frac{1}{8}$.

46. (d) $-2\pi < \alpha - \beta < 2\pi$

$\cos(\alpha-\beta) = 1 \Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta$

$\cos 2\alpha = \frac{1}{e}$ and $-2\pi < 2\alpha < 2\pi$

Hence, there will be four solutions.

47. (b) Given that $A + B = \frac{\pi}{4} \Rightarrow \tan(A+B) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A\tan B} = 1$

$\Rightarrow \tan A + \tan B + \tan A\tan B = 1$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$.

48. (d) $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$= \dfrac{2\left(\frac{1}{2}\cos 10^o - \frac{\sqrt{3}}{2}\sin 10^o\right)}{\frac{2}{2}\left(= \frac{\cos 10^o - \sqrt{3}\sin 10^o}{\sin 10^o\cos 10^o}\sin 10^o\cos 10^o\right)}$

$= \frac{4\sin(30^o - 10^o)}{\sin 20^o} = \frac{4\sin 20^o}{\sin 20^o} = 4$.

49. (a) $\sin\theta + \sin 3\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow 2\sin 2\theta\cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow \sin 2\theta(2\cos\theta + 1) = \sin\alpha$ .....(i)

Now $\cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$2\cos 2\theta\cos\theta + \cos 2\theta = \cos\alpha$

$\cos 2\theta(2\cos\theta + 1) = \cos\alpha$ .....(ii)

From (i) and (ii),

$\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \alpha/2$.

50. (b) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{\frac{1}{1+\frac{1}{2^x}} + \frac{1}{1+2^{x+1}}}{1 - \frac{1}{1+1/2^x}\frac{1}{1+2^{x+1}}}$

$\Rightarrow \tan(\alpha+\beta) = \frac{2^x + 2.2^{x+x} + 2^x + 1}{1 + 2^x + 2.2^x + 2.2^{x+x} - 2^x}$

$\Rightarrow \tan(\alpha+\beta) = 1 \Rightarrow \alpha + \beta = \frac{\pi}{4}$.

51. (d) $\cot(A-B) = \frac{1}{\tan(A-B)} = \frac{1 + \tan A\tan B}{\tan A - \tan B}$

$= \frac{1}{\tan A - \tan B} + \frac{\tan A\tan B}{\tan A - \tan B} = \frac{1}{x} + \frac{1}{y}$.

52. (c)

$\sin 12^o \sin 48^o \sin 54^o = \frac{1}{2}\left\{\cos 36^o - \cos 60^o\right\}\cos 36^o$

$=\frac{1}{2}\left[\frac{\sqrt{5}+1}{4}-\frac{1}{2}\right]\left[\frac{\sqrt{5}+1}{4}\right]=\frac{1}{2}\left[\frac{\sqrt{5}-1}{4}\right]\left[\frac{\sqrt{5}+1}{4}\right]$

$=\frac{5-1}{32}=\frac{4}{32}=\frac{1}{8}$.

53. (a) Given that $\cos A = m\cos B \Rightarrow \frac{m}{1}=\frac{\cos A}{\cos B}$

$\Rightarrow \frac{m+1}{m-1}=\frac{\cos A+\cos B}{\cos A-\cos B}=\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)}$

$=\cot\left(\frac{A+B}{2}\right)\cot\left(\frac{B-A}{2}\right)$

Hence, $\cot\left(\frac{A+B}{2}\right)=\frac{m+1}{m-1}\tan\frac{B-A}{2}$.

54. (b) $\cos A+\cos(240^o+A)+\cos(240^o-A)$

$=\cos A+2\cos 240^o\cos A$

$=\cos A\{1+2\cos(180^o+60^o)\}=\cos A\left\{1+2\left(-\frac{1}{2}\right)\right\}$

$=0$.

55. (b) $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$

$\Rightarrow \frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)}=\frac{(a+b)+(a-b)}{(a+b)-(a-b)}$

$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y}=\frac{2a}{2b}\Rightarrow \frac{\tan x}{\tan y}=\frac{a}{b}$.

56. (d) $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$

$=\frac{\sin 2^4\frac{2\pi}{15}}{2^4\sin\frac{2\pi}{15}}=\frac{\sin\frac{32\pi}{15}}{16\sin\frac{2\pi}{15}}=\frac{1}{16}\frac{\sin\frac{2\pi}{15}}{\sin\frac{2\pi}{15}}=\frac{1}{16}$.

57. (b) $\cos 2B=\frac{\cos(A+C)}{\cos(A-C)}=\frac{\cos A\cos C-\sin A\sin C}{\cos A\cos C+\sin A\sin C}$

$\Rightarrow \frac{1-\tan^2 B}{1+\tan^2 B}=\frac{1-\tan A\tan C}{1+\tan A\tan C}$

$\Rightarrow 1+\tan^2 B-\tan A\tan C-\tan A\tan C\tan^2 B$

$=1-\tan^2 B+\tan A\tan C-\tan A\tan C\tan^2 B$

$\Rightarrow 2\tan^2 B=2\tan A\tan C\Rightarrow \tan^2 B=\tan A\tan C$

Hence, $\tan A$, $\tan B$ and $\tan C$ will be in G.P.

58. (a) $\left(\frac{\sin 2A}{1+\cos 2A}\right)\left(\frac{\cos A}{1+\cos A}\right)$

$=\frac{2\sin A\cos A}{2\cos^2 A}\frac{\cos A}{1+\cos A}=\frac{\sin A}{1+\cos A}=\tan\frac{A}{2}$.

59. (c) $\sqrt{2+\sqrt{2+2\cos 4\theta}}=\sqrt{2+\sqrt{2.2\cos^2 2\theta}}$

$=\sqrt{2+2\cos 2\theta}=\sqrt{4\cos^2\theta}=2\cos\theta$.

60. (a) $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2$

$=\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta+\sin^2\alpha+$
$\sin^2\beta+2\sin\alpha\sin\beta$

$=2\{1+\cos(\alpha-\beta)\}=4\cos^2\left(\frac{\alpha-\beta}{2}\right)$.